Introduction to Computer Science
Jacobs University Bremen
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Course: CH08-320101
Date: 2017-10-20
Due: 2017-10-31

ICS Problem Sheet #6

Problem 6.1: *completeness of $\rightarrow$ and $\neg$* (2 points)

Proof that the two elementary boolean functions $\rightarrow$ (implication) and $\neg$ (negation) are universal, i.e., they are sufficient to express all possible boolean functions.

Problem 6.2: *conjunctive and disjunctive normal form* (2+1+3 = 6 points)

Consider the following boolean formula:

$$\varphi(M, N, P, Q, R, S) = (\neg P \lor Q) \land (\neg Q \lor R) \land (\neg R \lor S) \land (\neg S \lor P) \land M \land \neg N$$

a) How many interpretations of the variables M, N, P, Q, R, S satisfy φ? Provide a proof for your answer. Hint: You may concentrate on sub-expressions and you can of course use truth tables.

b) Given the interpretations that satisfy φ, write the formula for φ in disjunctive normal form (DNF).

c) Using the equivalence laws for boolean expressions, derive the DNF representation of φ algebraically from the CNF representation. Write the derivation down step wise.

Problem 6.3: *tower of hanoi (haskell)* (2 points)

The Tower of Hanoi is a mathematical game or puzzle. It consists of three rods and a number of disks of different sizes, which can slide onto any rod. The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top, thus making a conical shape. The objective of the puzzle is to move the entire stack to another rod, obeying the following simple rules:

- Only one disk can be moved at a time.
- Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack.
- No disk may be placed on top of a smaller disk.

[Source: `https://en.wikipedia.org/wiki/Tower_of_Hanoi`]

Write a Haskell function with the signature

```
hanoi :: Int -> [Move]
```

that takes the number of discs and returns a list of moves that solves the problem of moving the tower from rod 1 to rod 3. A move is a pair (x, y) indicating the starting rod number x and the destination rod number y (move the top disk from x to y).

```
*Main> hanoi 1
[(1,3)]

*Main> hanoi 3
[(1,3),(1,2),(3,2),(1,3),(2,1),(2,3),(1,3)]
```

You have to explain how your Haskell function solves the problem. If your explanation does not match what your code is doing, you will not receive points.