Introduction to Computer Science
Jacobs University Bremen
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ICS 2018 Problem Sheet #4

Problem 4.1: *prefix order relations* (2+2+1 = 5 points)

Let Σ be a finite set (called an alphabet) and let Σ^* be the set of all words that can be created out the symbols in the alphabet Σ. (Σ^* is the Kleene closure of Σ, which includes the empty word ϵ.) A word $p \in \Sigma^*$ is called a prefix of a word $w \in \Sigma^*$ if there is a word $q \in \Sigma^*$ such that $w = pq$. A prefix p is called a proper prefix if $p \neq w$.

a) Let $\preceq \subseteq \Sigma^* \times \Sigma^*$ be a relation such that $p \preceq w$ for $p, w \in \Sigma^*$ if p is a prefix of w. Show that $\preceq$ is a partial order.

b) Let $\prec \subset \Sigma^* \times \Sigma^*$ be a relation such that for $p \prec w$ for $p, w \in \Sigma^*$ if p is a proper prefix of w. Show that $\prec$ is a strict partial order.

c) Are the two order relations $\preceq$ and $\prec$ total?

Make sure you write complete proofs for the properties of the order relations. Do not assume something is 'obvious' or 'trivial' — always reason with the definition of the order relation.

Problem 4.2: *function composition* (2+1+1 = 4 points)

Let A, B and C be sets and let $f : A \to B$ and $g : B \to C$ be two functions.

a) Prove the following statement: If $g \circ f$ is bijective, then f is injective and g is surjective.

b) Find an example demonstrating that $g \circ f$ is not bijective even though f is injective and g is surjective.

c) Find an example demonstrating that $g \circ f$ is bijective even though f is not surjective and g is not injective.

Problem 4.3: *suffixes and prefixes (haskell)* (1 point)

Implement the function `suffixes :: [a] -> [[a]]`, which returns the list of all proper suffixes of its argument, longest first. The argument is not a proper suffix of itself.

Implement the function `prefixes :: [a] -> [[a]]`, which returns the list of all proper prefixes of its argument, shortest first. The argument is not a proper prefix of itself.

Note that the empty string is both a proper prefix and a proper suffix.

```
> suffixes "123"
["23","3",""]
> suffixes "1"
[""]
> prefixes "123"
["","1","12"]
> prefixes "1"
[""]
```

Explain how your function works. Submit your Haskell code as a plain text file.