Introduction to Computer Science
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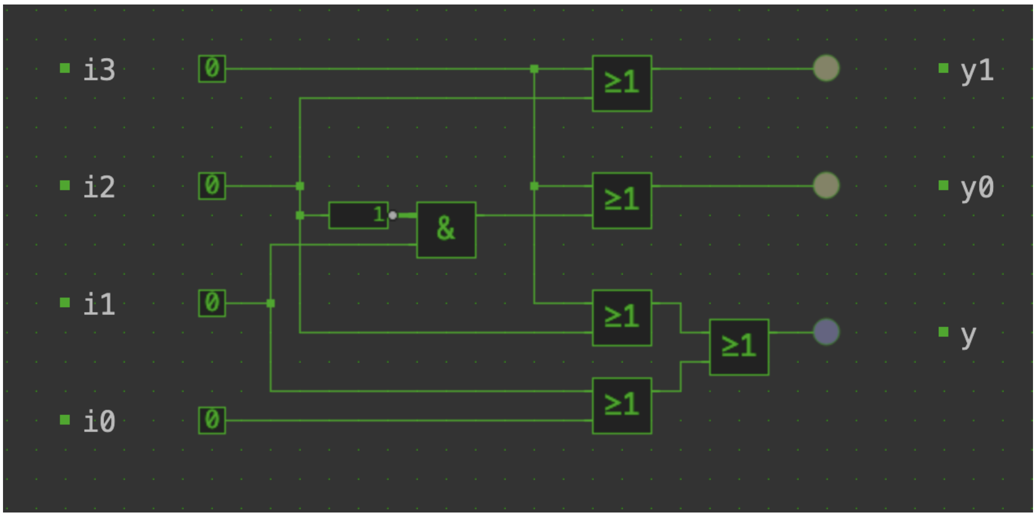
Module: CH-232
Date: 2021-10-29
Due: 2021-11-05

ICS 2021 Problem Sheet #8

Problem 8.1: *digital circuit analysis* (1+1+2 = 4 points)

You are given the following digital circuit. The circuit may as well be found online at http://simulator.io/board/pu8qlKwg1J/3 (but there is no guarantee that it persists).



a) Write down the truth table defining the outputs y_0, y_1, and y.

b) Write down the boolean expressions defining y_0, y_1, and y.

c) Describe in your own words what the circuit is doing and how it might be used.

Problem 8.2: *fold function duality theorems* (2+2+2 = 6 points)

The fold functions compute a value over a list (or some other type that is foldable) by applying an operator to the list elements and a neutral element. The foldl function assumes that the operator is left associative, the foldr function assumes that the operatore is right associative. For example, the function application

```
foldl (+) 0 [3,5,2,1]
```

results in the computation of ((((0+3)+5)+2)+1) and the function application

```
foldr (+) 0 [3,5,2,1]
```

results in the computation of (3+(5+(2+(1+0)))). The value computed by the fold functions may be more complex than a simple scalar. It is very well possible to construct a new list as part of the fold. For example:

```
map' :: (a -> b) -> [a] -> [b]
map' f xs = foldr ((:) . f) [] xs
```

The evaluation of `map' succ [1,2,3]` results in the list `[2,3,4]`. There are several duality theorems that can be stated for fold functions. Prove the following three duality theorems:

a) Let `op` be an associative operation with `e` as the neutral element:

> `op` is associative: `(x op y) op z = x op (y op z)`
> `e` is neutral element: `e op x = x` and `x op e = x`

Then the following holds for finite lists `xs`:

```
foldr op e xs = foldl op e xs
```

b) Let `op1` and `op2` be two operations for which

```
x `op1` (y `op2` z) = (x `op1` y) `op2` z
x `op1` e           = e `op2` x
```

holds. Then the following holds for finite lists `xs`:

```
foldr op1 e xs = foldl op2 e xs
```

c) Let `op` be an associative operation and `xs` a finite list. Then

```
foldr op a xs = foldl op' a (reverse xs)
```

holds with

```
x op' y = y op x
```