

## ICS 2022 Problem Sheet #2

**Problem 2.1:** *proof by contrapositive* (4 points)

Let  $x$  and  $y$  be real numbers, i.e.,  $x, y \in \mathbb{R}$ . If  $y^3 + yx^2 \leq x^3 + xy^2$ , then  $y \leq x$ .

**Problem 2.2:** *proof by induction* (4 points)

Let  $n$  be a natural number with  $n \geq 1$ . Prove that the following holds:

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \sum_{k=1}^n (2k-1)^2 = \frac{2n(2n-1)(2n+1)}{6}$$

**Problem 2.3:** *sum of divisors in haskell* (1+1 = 2 points)

The sum of divisors function  $\sigma_z(n)$  is defined as the sum over all divisors of a number  $n$  taken to the power of  $z$ . The function  $\sigma_z(n)$  can be more formally defines as

$$\sigma_z(n) = \sum_{d|n} d^z$$

where  $d|n$  is a shorthand for “ $d$  divides  $n$ ”. We implement this function in two steps.

- a) Write a function `divisors :: Int -> [Int]` that returns the list of divisors of a given positive integer  $n$ . The list of divisors includes 1 and the number  $n$  itself. For example:

```
ghci > divisors 1
[1]
ghci > divisors 6
[1,2,3,6]
ghci > divisors 15
[1,3,5,15]
```

Consider to define your function using a list comprehension. Here is a template to get started. Replace `undefined` with a suitable list comprehension.

```
-- Return the list of positive divisors of an integer n.
divisors :: Int -> [Int]
divisors n = undefined
```

Recall that the Haskell function `div` gives you the result of an integer division (truncated toward negative infinity) and the function `mod` gives you the integer modulus (remainder of an integer division).

- b) Write a function `sigma :: Int -> Int -> Int` that takes the two arguments  $z$  and  $n$  and returns the sum of the  $z$ th powers of the positive divisors of  $n$ . You can use the `sum` function to calculate the sum of a list of numbers. Here is a template to get started. Replace `undefined` with a suitable list comprehension.

```
-- Return the sum of divisors of n taken to the power of z
sigma :: Int -> Int -> Int
sigma z n = sum undefined
```

Some sample results:

```
ghci > sigma 0 12
6
ghci > sigma 1 12
28
```