Introduction to Computer Science
Jacobs University Bremen
Dr. Jürgen Schönwälder

Module: CH-232
Date: 2022-09-23
Due: 2022-09-30

ICS 2022 Problem Sheet #3

Problem 3.1: *cartesian products* (1+1 = 2 points)

Prove or disprove the following two propositions.

a) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

b) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

Problem 3.2: *reflexive, symmetric, transitive* (3 points)

For each of the following relations, determine whether they are reflexive, symmetric, or transitive. Provide a reasoning.

a) The absolute difference of the integer numbers a and b is less than or equal to 3.

$$R = \{\, (a,b) \mid a,b \in \mathbb{Z} \wedge |a - b| \le 3 \,\}$$

b) The last digit of the decimal representation of the integer numbers a and b is the same.

$$R = \{\, (a,b) \mid a,b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10) \,\}$$

Problem 3.3: *total, injective, surjective, bijective functions* (1+1 = 2 points)

Are the following functions total, injective, surjective, or bijective? Expain why or why not.

a) $f : \mathbb{N} \mapsto \mathbb{N}$ with $f(x) = 2x^2$

b) $f : \mathbb{R} \mapsto \mathbb{R}$ with $f(x) = x^2 + 6$

Problem 3.4: *types (haskell)* (1+2 = 3 points)

a) What is the type signature of the `zip` function? How many type variables appear in the type signature? Could it be more or less? Explain

b) What are the types of the following expressions? Explain why!

```
2 + 3
2 + 9 `div` 3
2 + 9 / 3
2 + sqrt 9
```