

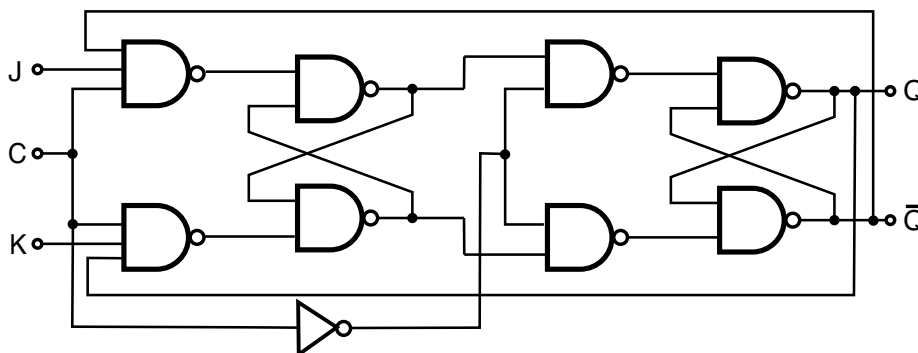
ICS 2022 Problem Sheet #9

Problem 9.1: *JK flip-flops*

(1+1+1+1 = 4 points)

JK flip-flops, also colloquially known as jump/kill flip-flops, augment the behaviour of SR flip-flops. The letters J and K were presumably picked by Eldred Nelson in a patent application.

The sequential digital circuit shown below shows the design of a JK flip-flop based on two SR NAND latches. Assume the circuit's output is $Q = 0$ and that the inputs are $J = 0$ and $K = 0$, and that the clock input is $C = 0$. (You can make use of the fact that we already know how an SR NAND latch behaves.)



- Suppose J transitions to 1 and C transitions to 1 soon after. Create a copy of the drawing and indicate for each line whether it carries a 0 or a 1.
- Some time later, C transitions back to 0 and soon after J transitions to 0 as well. Create another copy of the drawing and indicate for each line whether it carries a 0 or a 1.
- Some time later, J and K both transition to 1 and C transitions to 1 soon after. Create another copy of the drawing and indicate for each line whether it carries a 0 or a 1.
- Finally, C transitions back to 0 and soon after J and K both transition to 0 as well. Create another copy of the drawing and indicate for each line whether it carries a 0 or a 1.

Problem 9.2: *fold function duality theorems*

(2+2+2 = 6 points)

The fold functions compute a value over a list (or some other type that is foldable) by applying an operator to the list elements and a neutral element. The foldl function assumes that the operator is left associative, the foldr function assumes that the operator is right associative. For example, the function application

```
1 foldl (+) 0 [3,5,2,1]
```

results in the computation of $((((0+3)+5)+2)+1)$ and the function application

```
1 foldr (+) 0 [3,5,2,1]
```

results in the computation of $(3+(5+(2+(1+0))))$. The value computed by the fold functions may be more complex than a simple scalar. It is very well possible to construct a new list as part of the fold. For example:

```

1  map' :: (a -> b) -> [a] -> [b]
2  map' f xs = foldr (:) . f [] xs

```

The evaluation of `map' succ [1,2,3]` results in the list `[2,3,4]`. There are several duality theorems that can be stated for fold functions. Prove the following three duality theorems:

a) Let `op` be an associative operation with `e` as the neutral element:

`op` is associative: $(x \text{ op } y) \text{ op } z = x \text{ op } (y \text{ op } z)$
`e` is neutral element: $e \text{ op } x = x$ and $x \text{ op } e = x$

Then the following holds for finite lists `xs`:

$$\text{foldr } op \ e \ xs = \text{foldl } op \ e \ xs$$

b) Let `op1` and `op2` be two operations for which

$$\begin{aligned} x \text{ `op1` } (y \text{ `op2` } z) &= (x \text{ `op1` } y) \text{ `op2` } z \\ x \text{ `op1` } e &= e \text{ `op2` } x \end{aligned}$$

holds. Then the following holds for finite lists `xs`:

$$\text{foldr } op1 \ e \ xs = \text{foldl } op2 \ e \ xs$$

c) Let `op` be an associative operation and `xs` a finite list. Then

$$\text{foldr } op \ a \ xs = \text{foldl } op' \ a \ (\text{reverse } xs)$$

holds with

$$x \text{ op' } y = y \text{ op } x$$