

Problem Sheet #3

Problem 3.1: *set distributivity laws* (2 points)

Prove the following distributivity law for sets.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Problem 3.2: *cartesian products* (1+1 = 2 points)

Prove or disprove the following two propositions.

- a) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$
- b) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

Problem 3.3: *reflexive, symmetric, transitive* (3 points)

For each of the following relations, determine whether they are reflexive, symmetric, or transitive. Provide a reasoning.

- a) The absolute difference of the integer numbers a and b is less than or equal to 3.

$$R = \{ (a, b) \mid a, b \in \mathbb{Z} \wedge |a - b| \leq 3 \}$$

- b) The last digit of the decimal representation of the integer numbers a and b is the same.

$$R = \{ (a, b) \mid a, b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10) \}$$

Problem 3.4: *types (haskell)* (1+2 = 3 points)

- a) What is the type signature of the `zip` function? How many type variables appear in the type signature? Could it be more or less? Explain
- b) What are the types of the following expressions? Explain why!

```
2 + 3
2 + 9 `div` 3
2 + 9 / 3
2 + sqrt 9
```