Mathematical Foundations of Computer Science
Constructor University
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Module: CH-233
Date: 2024-09-27
Due: 2024-10-04

Problem Sheet #4

Problem 4.1: *reflexive, symmetric, transitive* (5 points)

For each of the following relations, determine whether they are reflexive, symmetric, or transitive. Provide a reasoning.

a) The absolute difference of the integer numbers a and b is less than or equal to 5.

$$R = \{\, (a, b) \in \mathbb{Z} \times \mathbb{Z} \mid |a - b| \leq 5 \,\}$$

b) The last digit of the decimal representation of the integer numbers a and b is the same.

$$R = \{\, (a, b) \in \mathbb{Z} \times \mathbb{Z} \mid (a \bmod 10) = (b \bmod 10) \,\}$$

c) The product of two positive natural numbers is a square number.

$$R = \{\, (a, b) \in \mathbb{N}^{+} \times \mathbb{N}^{+} \mid \exists n \in \mathbb{N} : ab = n^2 \,\}$$

Problem 4.2: *product order* (3 points)

The product relation $R = R_1 \times R_2$ of two relations R_1 and R_2 is defined as follows:

(i) $dom(R) = dom(R_1) \times dom(R_2)$

(ii) $codom(R) = codom(R_1) \times codom(R_2)$

(iii) $(a_1, b_1)R(a_2, b_2) \leftrightarrow a_1 R_1 a_2 \wedge b_1 R_2 b_2$

Prove the following statement: If R_1 and R_2 are both partial orders, then $R = R_1 \times R_2$ is a partial order as well.

Problem 4.3: *injective, surjective, bijective functions* (1+1 = 2 points)

Are the following functions injective, surjective, or bijective? Expain why or why not.

a) $f : \mathbb{N} \to \mathbb{N}$ with $f(x) = 2x + 1$

b) $f : \mathbb{N} \to \mathbb{N}$ with $f(x) = \begin{cases} x + 1 & \text{if } x \text{ is odd} \\ x - 1 & \text{if } x \text{ is even} \end{cases}$