Mathematical Foundations of Computer Science
Constructor University
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Module: CH-233
Date: 2024-10-18
Due: 2024-10-25

Problem Sheet #7

Problem 7.1: *implication and exclusive-or are universal* (3 points)

Prove that the combination of the two functions $\rightarrow$ and $\veebar$ is functionally complete. For each of the classic Boolean functions $\wedge$, $\vee$, and $\neg$, provide an equivalent Boolean expression using only $\rightarrow$ and $\veebar$. Show that the expressions are correct.

Problem 7.2: *espresso after lunch* (1+2+1+3 = 7 points)

Alice, Bob and Carol often have lunch together, but we do not know who likes to have an espresso after lunch. However, we know the following:

1. If Alice drinks an espresso, then so does Bob.
2. Bob and Carol never drink espresso together.
3. Alice or Carol drink an espresso (alone or together).
4. If Carol drinks an espresso, then so does Alice.

We introduce three boolean variables: The boolean variable a is true if Alice enjoys an espresso, the boolean variable b is true if Bob enjoys an espresso, and the boolean variable c is true if Carol enjoys an espresso.

a) Provide a boolean formula using only $\wedge, \vee, \neg$ for the boolean function $e(a, b, c)$ capturing the above rules.

b) Construct a truth table showing all interpretations of $e(a, b, c)$. Break things into meaningful steps so that we can award partial points in case things go wrong somewhere.

c) Out of the truth table, derive a simpler boolean formula defining $e(a, b, c)$, which is a conjunction (a logical and) of the three variables or their negation.

d) Take the boolean formula from a) and, using the equivalence laws introduced in class, algebraically derive the simpler boolean formula from c). Annotate each step of your derivation with the equivalence law that you apply so that we can follow along.