Mathematical Foundations of Computer Science Module: CH-233
Constructor University Date: 2024-11-01
Dr. Jürgen Schönwälder Due: 2024-11-08

Problem Sheet #9

Problem 9.1: *models satisfying a predicate logic formula* (1+1 = 2 points)

Consider a predicate r with two arguments and the formula $\varphi = \forall x \, \forall y \, \exists z \, ((r \, x \, y) \rightarrow (r \, y \, z))$.

a) Does $M \vDash \varphi$ hold for a model M with the domain $\mathcal{D} = \{a, b, c, d\}$ and the interpretation $\mathcal{I}(r) = \{(b, c), (b, b), (b, a)\}$? Explain.

b) Does $M \vDash \varphi$ hold for a model M with the domain $\mathcal{D} = \{a, b, c\}$ and the interpretation $\mathcal{I}(r) = \{(b, c), (a, b), (c, b)\}$? Explain.

Problem 9.2: *convert english statements into formulas* (3 points)

Let $\mathcal{D} = \{a, b\}^*$ be a domain. Let c be a constant, $f : \mathcal{D} \rightarrow \mathcal{D}$ be a function, and p, q be two predicates with the interpretation $\mathcal{I}$ defined below:

$$\begin{aligned} \mathcal{I}(c) &= aba \\ \mathcal{I}(f(x, y)) &= xy && \text{the concatenation of } x \text{ and } y \\ \mathcal{I}(p) &= \{w \in \mathcal{D} | w \text{ contains exactly one } a\} \\ \mathcal{I}(q) &= \{w \in \mathcal{D} | \text{length of } w \text{ is even}\} \end{aligned}$$

Translate the following sentences into predicate logic formulas.

a) The length of the word aba is odd.

b) There is a word containing a single a with even length.

c) The word aba contains exactly two a's.

d) The string x is a substring of y.

e) Every concatenation of words with even length yields a word with even length.

f) The string x is the empty word.

Problem 9.3: *prenex normal form* (2 points)

Convert the following formulas into prenex normal form.

$$\exists z \, ((\exists y \, (p \, y \, z)) \rightarrow (\forall y \, (q \, y \, z)))$$

Problem 9.4: *happy dragons* (2+1 = 3 points)

You are given three statements about dragons:

(1) Every dragon is happy when all its children can fly.

(2) All green dragons can fly.

(3) A dragon is green when it is a child of at least one green dragon.

a) Define suitable predicates and state their semantics and formalize the three statements in predicate logic.

b) Is there a simpler characterization of happy dragons? Explain.