# Problem Sheet #10

**Problem 10.1:** *product group* (2 points)

Let $G = (S_G, \star, e_G)$ and $H = (S_H, *, e_H)$ be two groups. Then the product of $G$ and $H$, denoted by $G \times H$, consists of all ordered pairs $(x, y)$ in $S_G \times S_H$. We define the operation $\circ$ on $S_G \times S_H$ as follows:

$$(x, y) \circ (x', y') = (x \star x', y * y')$$

Show that $G \times H = (S_G \times S_H, \circ, e)$ is a group for a suitable identity element $e$.

**Problem 10.2:** *subgroups of* $(\mathbb{Z}_6, +, 0)$ (3 points)

Determine all subgroups of $G = (\mathbb{Z}_6, +, 0)$ where $+$ denotes addition modulo $6$.

**Problem 10.3:** *another subgroup test* (5 points)

Let $G = (S, \circ, e)$ be a group and let $U$ be a non-empty subset of $S$. Show that $(U, \circ, e)$ is a subgroup of $G$ if and only if for all $a, b \in U$ the element $a \circ b^{-1} \in U$.