

**Problem Sheet #11**

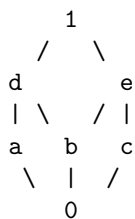
**Problem 11.1:** *lattices and sublattices*

(3+1+2+1 = 7 points)

Let  $L = (S, \sqcup, \sqcap)$  be a lattice. The lattice  $K = (S', \sqcup, \sqcap)$  is called a sublattice of  $L$  if  $S'$  is a non-empty subset of  $S$  and the following closure property holds:

$$\forall x, y \in S' : (x \sqcup y) \in S' \wedge (x \sqcap y) \in S'$$

Consider the following Hasse diagram:



- Show that this Hasse diagram represents a lattice.
- Is the lattice represented by the Hasse diagram distributive? Proof why or why not.
- Determine all elements of the lattice that have a complement.
- Does the set of all elements having a complement form a sublattice? Explain why or why not.

**Problem 11.2:** *non-isomorphic graphs with four vertices*

(1 point)

Set  $V$  be a set of four vertices. How many different graphs  $G_i = (V, E_i)$  exist that are not isomorphic? Explain or consider drawing all possible non-isomorphic graphs.

**Problem 11.3:** *graphs and degrees*

(1+1 = 2 points)

Let  $G = (V, E)$  be a graph.

- Show that the number of vertices with an odd degree is even.
- Show that if  $|V| > 1$ , then  $G$  has two vertices with the same degree.