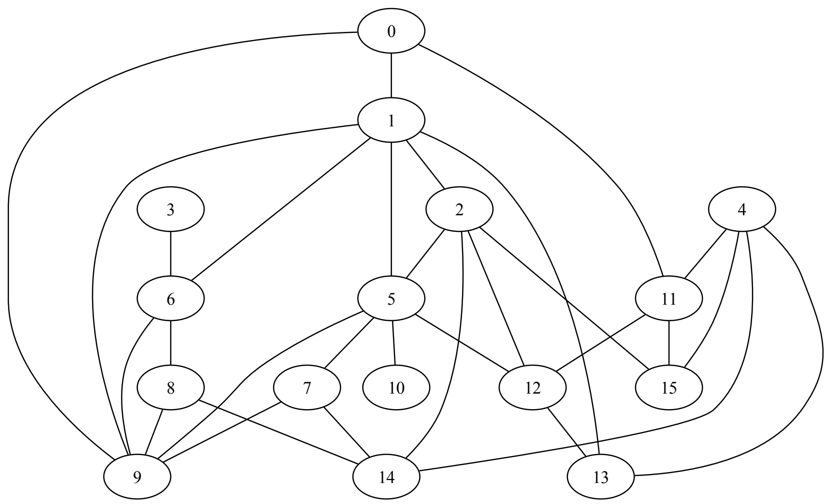
Mathematical Foundations of Computer Science
Constructor University
Dr. Jürgen Schönwälder

Module: CH-233
Date: 2024-11-22
Due: 2024-11-29

Problem Sheet #12

Problem 12.1: *graph traversals* (1+1 = 2 points)

The breadth-first and depth-first graph traversal algorithms generate spanning trees rooted at the start node by the way nodes are visited from other nodes. Whenever a node is added to the queue or stack, it is sufficient to remember the edge from the current node to the added node to build the spanning tree. Consider the following graph G.

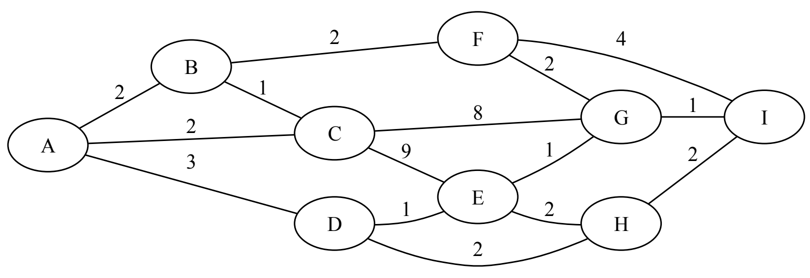


a) In which sequence are the nodes visited in a breadth-first traversal starting at node 0? Draw the resulting spanning tree. Explore neighbors according to the order of the node numbers.

b) In which sequence are the nodes visited in the depth-first traversal starting at node 0? Draw the resulting spanning tree. Explore neighbors according to the order of the node numbers.

Problem 12.2: *dijkstra shortest path algorithm* (4 points)

Consider the following graph G.

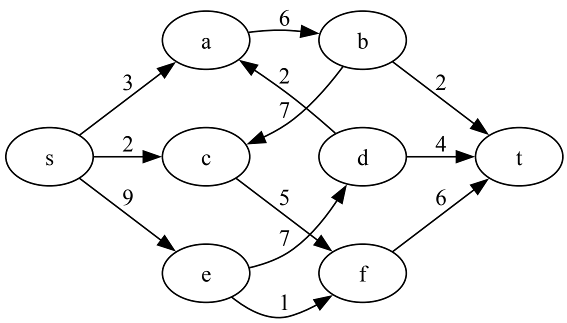


Execute Dijkstra's shortest paths algorithm from the start node A to dermine the shortest paths to all other nodes in the graph. Add a row to the following table at the end each loop iteration. The initial row shows the state after the initialization (i.e., before the loop starts).

Priority Queue	Distance									Predecessor									Visited
	A	B	C	D	E	F	G	H	I	A	B	C	D	E	F	G	H	I	
(A 0)	0	∞	∞	∞	∞	∞	∞	∞	∞										{}
⋮																			

Problem 12.3: *maximum flows and minimum capacity cuts* (3+1 = 4 points)

Consider the following network $N = (G, s, t, c)$ with the graph G, the source node s, the sink node t, and the capacity function c. The numbers at each edge indicate the capacity of the edge.



a) Manually execute the Ford–Fulkerson algorithm to determine the maximum flow. For each step, show which path you select, the resulting flow network and the derived residual network.

b) Detemine a cut S with minimum capacity.