Mathematical Foundations of Computer Science Module: CH-233
Constructor University Date: 2025-01-08
Dr. Jürgen Schönwälder Due: 2025-01-15

Problem Sheet #13

This sheet is only for students who failed to obtain the module achievement.

Problem 13.1: *sum formula* (2 points)

Prove that $1 + 4 + \ldots + (3n - 2) = \frac{1}{2}n(3n - 1)$ for $n \in \mathbb{N}$ and $n > 0$.

Problem 13.2: *equivalence relation* (2 points)

Let $A = \mathbb{N}_+ \times \mathbb{N}_+$ be the set of pairs of positive natural numbers. Let $\sim \subseteq A \times A$ be a binary relation on A. The tuple $((a, b), (c, d))$ is an element of $\sim$ if and only if $ad = bc$ (the product of a and d is equal to the product of b and c).

Show that $\sim$ is an equivalence relation (i.e., $\sim$ is reflexive, symmetric and transitive). For each property, first state what you are trying to show before you provide the argument.

Problem 13.3: *not-or is a universal boolean function* (3 points)

Prove that not-or ($\overline{\vee}$) is a universal boolean function by showing how $\overline{\vee}$ functions can implement the classic universal Boolean functions $\wedge, \vee, \neg$.

Problem 13.4: *IEEE 754 floating point numbers* (2 points)

The four hexadecimal bytes 0x47 0xf1 0x20 0x60 represent a floating point number (in big endian format). What is the floating point number in decimal notation? Explain.

Problem 13.5: *algebraic groups* (1 point)

Consider the finite set $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ with addition modulo 5 and multiplication modulo 5. Is $(\mathbb{Z}_5, \cdot)$ a group? Is $(\mathbb{Z}_5 \setminus \{0\}, \cdot)$ a group? Explain.