Mathematical Foundations of Computer Science Module: CH-233
Constructor University Date: 2025-09-12
Dr. Jürgen Schönwälder Due: 2025-09-19

Problem Sheet #2

Problem 2.1: *boyer moore algorithm* (2+2 = 4 points)

You have implemented a run and jump game, which is controlled by a game controller sending a sequence of run (R), jump (J), wait (W), and turn (T) control signals. A game play thus can be expressed with a sequence such as RRJRJRWRWRRJRWR. Using the Boyer Moore algorithm, we can determine whether a sequence of a game play contains certain control sequences.

Let $\Sigma = \{J, R, T, W\}$ be an alphabet and $t \in \Sigma^*$ be a text of length n describing a game play. Let $p \in \Sigma^*$ be a pattern of length m. We are looking for the first occurrence of p in t.

Consider the text $t = RTJRJRWRWRTJRTJRWR$ and the pattern $p = JRTJR$.

a) Execute the Boyer-Moore string search algorithm with the good suffix rule only. How many alignments are used? How many comparisons are done?

b) Execute the Boyer-Moore string search algorithm with the bad character rule and the good suffix rule. How many alignments are used? How many comparisons are done?

Problem 2.2: *landau sets* (2+2+2 = 6 points)

Let $\sin : \mathbb{R} \to \mathbb{R}$ denote the sine trigonometric function and let $n! : \mathbb{N} \to \mathbb{N}$ denote the factorial of n, that is $n! = \prod_{k=1}^{n} k = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$.

a) Let $f : \mathbb{N} \to \mathbb{R}$ be a function with $f(n) = n \cdot \sin(n^2)$. Show that $f \in O(n)$.

b) Let $f : \mathbb{N} \to \mathbb{N}$ be a function with $f(n) = 2^n$. Show that $f \notin \Theta(3^n)$.

c) Let $f : \mathbb{N} \to \mathbb{N}$ be a function with $f(n) = \sum_{i=0}^{n} i!$. Show that $f \in \Theta(n!)$.