Mathematical Foundations of Computer Science Module: CH-233
Constructor University Date: 2025-09-19
Dr. Jürgen Schönwälder Due: 2025-09-26

Problem Sheet #3

Problem 3.1: *proof by contrapositive* (4 points)

Let $x, y \in \mathbb{R}$ be real numbers. If $y^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$.

Problem 3.2: *proof by induction* (4 points)

Let $n \in \mathbb{N}$ be a natural number with $n \geq 1$. Prove by induction that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.

Problem 3.3: *cartesian products* (1+1 = 2 points)

Prove or disprove the following two propositions.

a) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

b) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$