Mathematical Foundations of Computer Science
Constructor University
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Module: CH-233
Date: 2025-09-26
Due: 2025-10-03

Problem Sheet #4

Problem 4.1: *properties of relations* (2+3 = 5 points)

For each of the following relations, determine whether they are reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. Provide a reasoning for each property why it holds or why it does not hold.

a) The absolute difference of the integer numbers a and b is less than or equal to 5.

$$R = \{ (a, b) \in \mathbb{Z} \times \mathbb{Z} \mid |a - b| \leq 5 \}$$

b) Let $m \bmod n$ denote the positive remainder of m divided by n with $n \neq 0$.

$$R = \{ (a, b) \in \mathbb{Z} \times \mathbb{Z} \mid (a \bmod 10) = (b \bmod 5) \}$$

Problem 4.2: *properties of relation composition* (1+1+1 = 3 points)

Let $R \subseteq X \times Y$ and $S \subseteq Y \times Z$ be two binary relations. We define the composition of R and S as follows:

$$R; S = \{ (x, z) \in X \times Z \mid \exists y \in Y. (x, y) \in R \wedge (y, z) \in S \}$$

Prove or disprove the following statements for the composition of binary endorelations:

a) If R and S are both reflexive on A, then $R; S$ is reflexive as well.

b) If R and S are both symmetric on A, then $R; S$ is symmetric as well.

c) If R and S are both transitive on A, then $R; S$ is transitive as well.

Problem 4.3: *injective, surjective, bijective functions* (1+1 = 2 points)

Are the following functions injective, surjective, or bijective? Expain why or why not.

a) $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x \cdot sin(x)$

b) $g \circ f : \mathbb{N} \to \mathbb{N}$ with $f(x) = x^3$ and $g(x) = 2x + 1$