Mathematical Foundations of Computer Science
Constructor University
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Module: CH-233
Date: 2024-10-17
Due: 2024-10-24

Problem Sheet #7

Problem 7.1: *not-or is a universal boolean function* (3 points)

Prove that not-or ($\overline{\vee}$) is a universal boolean function by showing how $\overline{\vee}$ functions can implement the classic universal Boolean functions $\wedge, \vee, \neg$.

Problem 7.2: *brandy after dinner* (1+2+1+3 = 7 points)

Leo, Mark and Nick often have dinner together, but we do not know who likes to have a brandy after the dinner. However, we know the following:

1. If Leo drinks a brandy, then so does Mark.
2. Mark and Nick never drink brandy together.
3. Leo or Nick drink a brandy (alone or together).
4. If Nick drinks a brandy, then so does Leo.

We introduce three boolean variables: The variable L is true if Leo enjoys a brandy, the variable M is true if Mark enjoys a brandy, and the variable N is true if Nick enjoys a brandy.

a) Provide a boolean formula using only $\wedge, \vee, \neg$ for the function $D(L, M, N)$ capturing the above rules.

b) Construct a truth table showing all interpretations of $D(L, M, N)$. Break things into meaningful steps so that we can award partial points in case things go wrong somewhere.

c) Out of the truth table, derive a simpler boolean formula defining $D(L, M, N)$, which is a conjunction (a logical and) of the three variables or their negation.

d) Take the boolean formula from a) and, using the laws introduced in class, algebraically derive the simpler boolean formula from c). Annotate each step of your derivation with the equivalence law that you apply so that we can follow along.