

### Problem Sheet #9

#### Problem 9.1: *louvre jewels*

(2+1 = 3 points)

Some trickster did steal some jewels from the Louvre in Paris. The investigations identified three possible tricksters,  $A$ ,  $B$ , and  $C$ . They may have stolen the jewels individually or in a group of two or three tricksters. The investigators obtained the following statements:

1. If  $A$  is not guilty, then  $B$  is guilty.
2. If  $B$  is not guilty, then both  $A$  and  $C$  are guilty.

The investigators know their sources. They believe that they can trust the first statement but the second statement is known to be false.

- a) Who did commit the crime? Prove your answer using Boolean algebra equivalence laws.
- b) Represent the statements and your result into an argument in propositional logic and write it down in sequent notation.

#### Problem 9.2: *valid and sound arguments*

(2 points)

Which of the following arguments are valid? Which ones are sound? Explain.

- a) Walnut trees are green:

All trees are green.  
A walnut tree is a tree.  
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A walnut tree is green.

- b) I spent a lot of my time learning:

If I spent a lot of my time learning, I will pass the exam.  
I have passed the exam.  
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I have spent a lot of my time learning.

- c) Someone is not good at logic:

If a person is a philosopher, the person is good at logic.  
This person is not a philosopher.  
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This person is not good at logic.

- d) Objects violating geometry:

If something is both square and circular, then it violates geometry.  
This object is both square and circular.  
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Therefore, this object violates geometry.

#### Problem 9.3: *models satisfying a predicate logic formula*

(1+1 = 2 points)

Consider a predicate  $r$  with two arguments and the formula  $\varphi = \forall x \forall y \exists z ((r \ x \ y) \rightarrow (r \ y \ z))$ .

- a) Does  $M \models \varphi$  hold for a model  $M$  with the domain  $\mathcal{D} = \{a, b, c, d\}$  and the interpretation  $\mathcal{I}(r) = \{(b, c), (b, b), (b, a)\}$ ? Explain.
- b) Does  $M \models \varphi$  hold for a model  $M$  with the domain  $\mathcal{D} = \{a, b, c\}$  and the interpretation  $\mathcal{I}(r) = \{(b, c), (a, b), (c, b)\}$ ? Explain.

**Problem 9.4:** *convert english statements into formulas*

(3 points)

Let  $\mathcal{D} = \{a, b\}^*$  be a domain. Let  $c$  be a constant,  $f : \mathcal{D} \rightarrow \mathcal{D}$  be a function, and  $p, q$  be two predicates with the interpretation  $\mathcal{I}$  defined below:

$$\begin{aligned}\mathcal{I}(c) &= aba \\ \mathcal{I}(f(x, y)) &= xy && \text{the concatenation of } x \text{ and } y \\ \mathcal{I}(p) &= \{w \in \mathcal{D} \mid w \text{ contains exactly one } a\} \\ \mathcal{I}(q) &= \{w \in \mathcal{D} \mid \text{length of } w \text{ is even}\}\end{aligned}$$

Translate the following sentences into predicate logic formulas.

- a) The length of the word  $aba$  is odd.
- b) There is a word containing a single  $a$  with even length.
- c) The word  $aba$  contains exactly two  $a$ 's.
- d) The string  $x$  is a substring of  $y$ .
- e) Every concatenation of words with even length yields a word with even length.
- f) The string  $x$  is the empty word.