

Problem Sheet #10

Problem 10.1: *prenex normal form*

(2 points)

Convert the following formulas into prenex normal form.

$$\exists z ((\exists y (p y z)) \rightarrow (\forall y (q y z)))$$

Problem 10.2: *happy dragons*

(2+1 = 3 points)

You are given three statements about dragons:

- (1) Every dragon is happy when all its children can fly.
- (2) All green dragons can fly.
- (3) A dragon is green when it is a child of at least one green dragon.

- a) Define suitable predicates and state their semantics and formalize the three statements in predicate logic.
- b) Is there a simpler characterization of happy dragons? Explain.

Problem 10.3: *product group*

(2 points)

Let $G = (S_G, \star, e_G)$ and $H = (S_H, *, e_H)$ be two groups. Then the product of G and H , denoted by $G \times H$, consists of all ordered pairs (x, y) in $S_G \times S_H$. We define the operation $\circ$ on $S_G \times S_H$ as follows:

$$(x, y) \circ (x', y') = (x \star x', y * y')$$

Show that $G \times H = (S_G \times S_H, \circ, e)$ is a group for a suitable identity element e .

Problem 10.4: *subgroups of $(\mathbb{Z}_6, +, 0)$*

(3 points)

Determine all subgroups of $G = (\mathbb{Z}_6, +, 0)$ where $+$ denotes addition modulo 6.