

Problem Sheet #11

Problem 11.1: *group of self-inverses is abelian*

(1 point)

Let $(G, \circ, e)$ be a group. Show that if $a^2 = e$ for all $a \in G$, then G is an abelian group.

Problem 11.2: *group of automorphisms with composition*

(2 points)

Let $G = (S, \star)$ be a group. Let A be the set of all group automorphisms $f : G \rightarrow G$. Show that $H = (A, \circ)$ is a group with $\circ$ representing the composition of automorphisms in A .

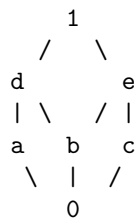
Problem 11.3: *lattices and sublattices*

(3+1+2+1 = 7 points)

Let $L = (S, \sqcup, \sqcap)$ be a lattice. The lattice $K = (S', \sqcup, \sqcap)$ is called a sublattice of L if S' is a non-empty subset of S and the following closure property holds:

$$\forall x, y \in S' : (x \sqcup y) \in S' \wedge (x \sqcap y) \in S'$$

Consider the following Hasse diagram:



- Show that this Hasse diagram represents a lattice.
- Is the lattice represented by the Hasse diagram distributive? Proof why or why not.
- Determine all elements of the lattice that have a complement.
- Does the set of all elements having a complement form a sublattice? Explain why or why not.