Mathematical Foundations of Computer Science
Constructor University
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Module: CH-233
Date: 2025-11-14
Due: 2025-11-21

Problem Sheet #11

Problem 11.1: *group of self-inverses is abelian* (1 point)

Let $(G, \circ, e)$ be a group. Show that if $a^2 = e$ for all $a \in G$, then G is an abelian group.

Problem 11.2: *group of automorphisms with composition* (2 points)

Let $G = (S, \star)$ be a group. Let A be the set of all group automorphisms $f : G \to G$. Show that $H = (A, \circ)$ is a group with $\circ$ representing the composition of automorphisms in A.

Problem 11.3: *lattices and sublattices* (3+1+2+1 = 7 points)

Let $L = (S, \sqcup, \sqcap)$ be a lattice. The lattice $K = (S', \sqcup, \sqcap)$ is called a sublattice of L if S' is a non-empty subset of S and the following closure property holds:

$$\forall x, y \in S' : (x \sqcup y) \in S' \land (x \sqcap y) \in S'$$

Consider the following Hasse diagram:

```
    1
  /   \
d       e
| \   / |
a   b   c
 \  |  /
    0
```

a) Show that this Hasse diagram represents a lattice.

b) Is the lattice represented by the Hasse diagram distributive? Proof why or why not.

c) Determine all elements of the lattice that have a complement.

d) Does the set of all elements having a complement form a sublattice? Explain why or why not.