

### Problem Sheet #13

**This sheet is only for students who failed to obtain the module achievement.**

**Problem 13.1:** *sum formula* (2 points)

Prove that  $1 + 4 + \dots + (3n - 2) = \frac{1}{2}n(3n - 1)$  for  $n \in \mathbb{N}$  and  $n > 0$ .

**Problem 13.2:** *equivalence relation* (2 points)

Let  $A = \mathbb{N}_+ \times \mathbb{N}_+$  be the set of pairs of positive natural numbers. Let  $\sim \subseteq A \times A$  be a binary relation on  $A$ . The tuple  $((a, b), (c, d))$  is an element of  $\sim$  if and only if  $ad = bc$  (the product of  $a$  and  $d$  is equal to the product of  $b$  and  $c$ ).

Show that  $\sim$  is an equivalence relation (i.e.,  $\sim$  is reflexive, symmetric and transitive). For each property, first state what you are trying to show before you provide the argument.

**Problem 13.3:** *not-or is a universal boolean function* (3 points)

Prove that not-or ( $\nabla$ ) is a universal boolean function by showing how  $\nabla$  functions can implement the classic universal Boolean functions  $\wedge, \vee, \neg$ .

**Problem 13.4:** *IEEE 754 floating point numbers* (2 points)

The four hexadecimal bytes 0x47 0xf1 0x20 0x60 represent a floating point number (in big endian format). What is the floating point number in decimal notation? Explain.

**Problem 13.5:** *algebraic groups* (1 point)

Consider the finite set  $S = \{0, 1, 2, 3, 4\}$  with multiplication modulo 5. Is  $(S, \cdot)$  a group? Is  $(S \setminus \{0\}, \cdot)$  a group? Explain.