

Problem Sheet #13

This sheet is only for students who failed to obtain the module achievement.

Problem 13.1: *sum formula*

(2 points)

Prove that $1 + 4 + \dots + (3n - 2) = \frac{1}{2}n(3n - 1)$ for $n \in \mathbb{N}$ and $n > 0$.

Problem 13.2: *equivalence relation*

(2 points)

Let $A = \mathbb{N}_+ \times \mathbb{N}_+$ be the set of pairs of positive natural numbers. Let $\sim \subseteq A \times A$ be a binary relation on A . The tuple $((a, b), (c, d))$ is an element of \sim if and only if $ad = bc$ (the product of a and d is equal to the product of b and c).

Show that \sim is an equivalence relation (i.e., \sim is reflexive, symmetric and transitive). For each property, first state what you are trying to show before you provide the argument.

Problem 13.3: *not-or is a universal boolean function*

(3 points)

Prove that not-or (∇) is a universal boolean function by showing how ∇ functions can implement the classic universal Boolean functions \wedge, \vee, \neg .

Problem 13.4: *IEEE 754 floating point numbers*

(2 points)

The four hexadecimal bytes 0x47 0xf1 0x20 0x60 represent a floating point number (in big endian format). What is the floating point number in decimal notation? Explain.

Problem 13.5: *algebraic groups*

(1 point)

Consider the finite set $S = \{0, 1, 2, 3, 4\}$ with multiplication modulo 5. Is (S, \cdot) a group? Is $(S \setminus \{0\}, \cdot)$ a group? Explain.