

Problem Sheet #2

Problem 2.1: *landau sets*

(2+2+2 = 6 points)

Let $\sin : \mathbb{R} \rightarrow \mathbb{R}$ denote the sine trigonometric function and let $n! : \mathbb{N} \rightarrow \mathbb{N}$ denote the factorial of n , that is $n! = \prod_{k=1}^n k = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$.

- a) Let $f : \mathbb{N} \rightarrow \mathbb{R}$ be a function with $f(n) = n \cdot \sin(n^2)$. Show that $f \in O(n)$.
- b) Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be a function with $f(n) = 2^n$. Show that $f \notin \Theta(3^n)$.
- c) Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be a function with $f(n) = \sum_{i=0}^n i!$. Show that $f \in \Theta(n!)$.

Problem 2.2: *proof by contrapositive*

(4 points)

Let $a \in \mathbb{Z}$ be an integer number. If a^{32} is an odd number, then a^4 is an odd number as well.