Mathematical Foundations of Computer Science Module: CH-233
Constructor University Date: 2026-10-23
Dr. Jürgen Schönwälder Due: 2026-10-30

Problem Sheet #7

Problem 7.1: *quine-mccluskey algorithm* (2+4+3+1 = 10 points)

Consider natural numbers in the range 0...63, which can be represented using six bits x_i with $i \in \{0,\ldots,5\}$. The boolean function $f(x_5,x_4,x_3,x_2,x_1,x_0)$ is true when the number $(x_5x_4x_3x_2x_1x_0)_2$ is a Fibonacci number and false otherwise. The Fibonacci sequence $F_0,F_1,F_2,\ldots$ is defined by the recurrance relation $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0$ and $F_1 = 1$.

a) Provide a boolean expression in DNF defining the boolean function f. What is the cost of the DNF expression?

b) Determine the prime implicants of the boolean function f.

c) Construct the prime implicant chart and determine the essential prime implicants. What is a minimal set of prime implicants covering the boolean function f?

d) Write out a minimal boolean expression defining f using mathematical logic notation. What is the cost of the minimal boolean expression?

For calculating the cost of a boolean expression, we only consider $\wedge$ and $\vee$ operations.