Mathematical Foundations of Computer Science Module: CH-233
Constructor University Date: 2026-10-30
Dr. Jürgen Schönwälder Due: 2026-11-06

Problem Sheet #8

Problem 8.1: *louvre jewels* (2+1 = 3 points)

Some trickster did steel some jewels from the Louvre in Paris. The investigations identified three possible tricksters, A, B, and C. They may have stolen the jewels individually or in a group of two or three tricksters. The investigators obtained the following statements:

1. If A is not guilty, then B is guilty.
2. If B is not guilty, then both A and C are guilty.

The investigators know their sources. They believe that they can trust the first statement but the second statement is known to be false.

a) Who did commit the crime? Prove your answer using Boolean algebra equivalence laws.

b) Represent the statements and your result into an argument in propositional logic and write it down in sequent notation.

Problem 8.2: *valid and sound arguments* (2 points)

Which of the following arguments are valid? Which ones are sound? Explain.

a) Walnut trees are green:

All trees are green.
A walnut tree is a tree.

A walnut tree is green.

b) I spent a lot of my time learning:

If I spent a lot of my time learning, I will pass the exam.
I have passed the exam.

I have spent a lot of my time learning.

c) Someone is not good at logic:

If a person is a philosopher, the person is good at logic.
This person is not a philosopher.

This person is not good at logic.

d) Objects violating geometry:

If something is both square and circular, then it violates geometry.
This object is both square and circular.

Therefore, this object violates geometry.

Problem 8.3: *models satisfying a predicate logic formula* (1+1 = 2 points)

Consider a predicate r with two arguments and the formula $\varphi = \forall x \, \forall y \, \exists z \, ((r \, x \, y) \rightarrow (r \, y \, z))$.

a) Does $M \vDash \varphi$ hold for a model M with the domain $\mathcal{D} = \{a, b, c, d\}$ and the interpretation $\mathcal{I}(r) = \{(b, c), (b, b), (b, a)\}$? Explain.

b) Does $M \vDash \varphi$ hold for a model M with the domain $\mathcal{D} = \{a, b, c\}$ and the interpretation $\mathcal{I}(r) = \{(b, c), (a, b), (c, b)\}$? Explain.

Problem 8.4: *convert english statements into formulas* (3 points)

Let $\mathcal{D} = \{a, b\}^*$ be a domain. Let c be a constant, $f : \mathcal{D} \to \mathcal{D}$ be a function, and p, q be two predicates with the interpretation $\mathcal{I}$ defined below:

$$\begin{aligned} \mathcal{I}(c) &= aba \\ \mathcal{I}(f(x,y)) &= xy && \text{the concatenation of } x \text{ and } y \\ \mathcal{I}(p) &= \{w \in \mathcal{D} | w \text{ contains exactly one } a\} \\ \mathcal{I}(q) &= \{w \in \mathcal{D} | \text{length of } w \text{ is even}\} \end{aligned}$$

Translate the following sentences into predicate logic formulas.

a) The length of the word aba is odd.

b) There is a word containing a single a with even length.

c) The word aba contains exactly two a's.

d) The string x is a substring of y.

e) Every concatenation of words with even length yields a word with even length.

f) The string x is the empty word.