Mathematical Foundations of Computer Science Module: CH-233
Constructor University Date: 2026-11-06
Dr. Jürgen Schönwälder Due: 2026-11-13

Problem Sheet #9

Problem 9.1: *happy dragons* (2+1 = 3 points)

You are given three statements about dragons:

(1) Every dragon is happy when all its children can fly.

(2) All green dragons can fly.

(3) A dragon is green when it is a child of at least one green dragon.

a) Define suitable predicates and state their semantics and formalize the three statements in predicate logic.

b) Is there a simpler characterization of happy dragons? Explain.

Problem 9.2: *prenex normal form* (2 points)

Convert the following formulas into prenex normal form.

$$\exists z\,((\exists y\,(p\,y\,z)) \rightarrow (\forall y\,(q\,y\,z)))$$

Problem 9.3: *product group* (2 points)

Let $G = (S_G, \star, e_G)$ and $H = (S_H, *, e_H)$ be two groups. Then the product of G and H, denoted by $G \times H$, consists of all ordered pairs (x, y) in $S_G \times S_H$. We define the operation $\circ$ on $S_G \times S_H$ as follows:

$$(x, y) \circ (x', y') = (x \star x', y * y')$$

Show that $G \times H = (S_G \times S_H, \circ, e)$ is a group for a suitable identity element e.

Problem 9.4: *subgroups of* $(\mathbb{Z}_6, +, 0)$ (3 points)

Determine all subgroups of $G = (\mathbb{Z}_6, +, 0)$ where $+$ denotes addition modulo 6.