Mathematical Foundations of Computer Science Module: CH-233
Constructor University Date: 2026-11-13
Dr. Jürgen Schönwälder Due: 2026-11-20

Problem Sheet #10

Problem 10.1: *group conjugation homomorphism* (1 point)

Let $(G, \circ, e)$ be a group and $g \in G$ be an element of G. Let $f : G \to G$ be a map over G with $f(a) = g \circ a \circ g^{-1}$ for all $a \in G$. Show that f is a group homomorphism.

Problem 10.2: *group of automorphisms with composition* (2 points)

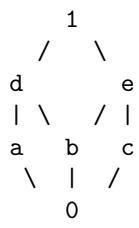
Let $G = (S, \star)$ be a group. Let A be the set of all group automorphisms $f : G \to G$. Show that $H = (A, \circ)$ is a group with $\circ$ representing the composition of automorphisms in A.

Problem 10.3: *lattices and sublattices* (3+1+2+1 = 7 points)

Let $L = (S, \sqcup, \sqcap)$ be a lattice. The lattice $K = (S', \sqcup, \sqcap)$ is called a sublattice of L if S' is a non-empty subset of S and the following closure property holds:

$$\forall x, y \in S' : (x \sqcup y) \in S' \wedge (x \sqcap y) \in S'$$

Consider the following Hasse diagram:



a) Show that this Hasse diagram represents a lattice.

b) Is the lattice represented by the Hasse diagram distributive? Proof why or why not.

c) Determine all elements of the lattice that have a complement.

d) Does the set of all elements having a complement form a sublattice? Explain why or why not.