

## SADS 2018 Problem Sheet #2

### Problem 2.1: *partial correctness of the gcd algorithm*

(4 points)

Prove step-by-step the partial correctness of the following program using Hoare Logic. The program calculates the greatest common divisor (gcd). You can assume  $\vdash \text{gcd}(a, 0) = a$  and  $\vdash \text{gcd}(a, b) = \text{gcd}(b, a)$ .

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**Precondition:**  $\{X = x \wedge Y = y \wedge x > 0 \wedge y > 0\}$

```
1: while  $Y \neq 0$  do  
2:    $Z := X \% Y$   
3:    $X := Y$   
4:    $Y := Z$   
5: od
```

**Postcondition:**  $\{X = \text{gcd}(x, y)\}$

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### Problem 2.2: *total correctness of the gcd algorithm*

(1+4+1 = 6 points)

Prove the total correctness of the following program by annotating the program and deriving the verification conditions (prove goals). The program calculates the greatest common divisor (gcd). You can assume  $\vdash \text{gcd}(a, 0) = a$  and  $\vdash \text{gcd}(a, b) = \text{gcd}(b, a)$ .

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**Precondition:**  $\{X = x \wedge Y = y \wedge x > 0 \wedge y > 0\}$

```
1: while  $Y \neq 0$  do  
2:    $Z := X \% Y$   
3:    $X := Y$   
4:    $Y := Z$   
5: od
```

**Postcondition:**  $\{X = \text{gcd}(x, y)\}$

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- Provide the annotated program.
- Derive the verification conditions.
- Prove the verification conditions.