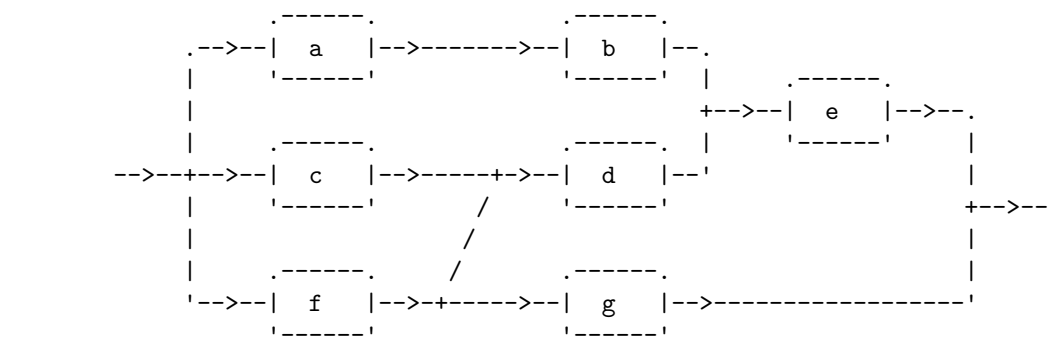
Secure and Dependable Systems Module: CO-566
Constructor University Date: 2023-02-09
Dr. Jürgen Schönwälder **Due: 2023-02-16**

Problem Sheet #1

Problem 1.1: *system reliability* (3+2+2 = 7 points)

Consider a computing system with the following component structure:



a) Assuming that failures are independent, derive a formula for the reliability of the system. You can write down the formula in either plain math or as function definitions in Haskell (using Haskell as a calculator here may be convenient).

b) Assuming all components have the same reliability, generate a plot showing the system reliability as a function of the component reliability. At which point does the system reliability become better than the component reliablity?

c) Assume all components have a reliability of 0.8. You have the financial resources to replace one component with a component that has a reliability of 0.95. Which component do you replace to maximize the system reliability? What is the new system reliability you can achieve and what is the improvement?

Problem 1.2: *k out of n systems* (1+1+1 = 3 points)

Let $R_i(t)$ denote the probability that component i works at time t and assume that component failures are independent. A serial system requires all n components to work and we know that the reliability of a serial system is given by

$$R_s(t) = \prod_i R_i(t).$$

A parallel system requires that at least one out of n components works and we know that the reliability of a parallel system is given by

$$R_p(t) = 1 - \prod_i \left(1 - R_i(t)\right).$$

The serial and the parallel systems can be seen as special cases of a general k out of n system where at least k components have to work in order for the overall system to work (a serial system is the special case where $k = n$ and a parallel system is the special case where $k = 1$).

a) Derive a general formula to calculate the reliability of a k out of n system.

b) Produce a formula for the special case where all components have the same reliability p.

c) Calculate how the probability of a system with $n = 5$ components changes if at least $k \in \{1, \ldots, 5\}$ components need to function. Assume that all components have a reliability of 80% (using Haskell as a calculator here may be convenient).