## Problem Sheet \#8

Problem 8.1: block encryption modes of operation
Consider a simple symmetric block cipher with a block size and a key size of 4 bits. The encryption function $E(k, m)$ is defined as

$$
E(k, m)=s(k \oplus m)
$$

where $k$ is the 4-bit key, $m$ is a 4-bit cleartext block, $\oplus$ is the bitwise exclusive-or operation and the function $s$ is a bijective substitution defined via the following table:

| $m$ | 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s(m)$ | 0010 | 1010 | 0110 | 1100 | 1001 | 0000 | 1110 | 0101 |
| $m$ | 1000 | 1001 | 1010 | 1011 | 1100 | 1101 | 1110 | 1111 |
| $s(m)$ | 0001 | 1000 | 0100 | 1111 | 0111 | 1101 | 0011 | 1011 |

Hint: In your solution, you can write + instead of $\oplus$ to refer to the exclusive-or operation.
a) Define the decryption function $D(k, c)$ and show that the decryption function is correct.
b) Encrypt the message 101000110101 with the key $k=1010$ using the Electronic Code Book (ECB) mode. Write the ciphertext in space-separated 4-bit blocks.
c) Encrypt the message 101000110101 with the key $k=1010$ using the Cipher Block Chaining (CBC) mode using the initialization vector $I V=1001$. Write the ciphertext in spaceseparated 4-bit blocks.
d) Encrypt the message 101000110101 with the key $k=1010$ using the Output Feedback Mode (OFB) using the initialization vector $I V=1001$. Write the ciphertext in space-separated 4-bit blocks.
e) Encrypt the message 101000110101 with the key $k=1010$ using the Counter Mode (CTR) using the two bit nonce $N=11$ (in binary) and a two bit counter. Write the ciphertext in space-separated 4-bit blocks.

